



## Effects of Cosine shape Symmetric Stenosis on the Blood Flow in an Artery

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### Abstract

Blood flow characteristic changes from its normal state to a distributed flow condition in the presence of a arteriosclerosis in the artery. A study has been carried out to analyze the non-Newtonian blood flow in a stenotic artery. For this study, a very common shape of stenosis, that is modified cosine, are modeled. The equations of motion of blood flow are solved analytically using appropriate boundary condition. The expressions for different flow parameters like axial velocity, wall shear stress and volumetric flow rate have been obtained. The effect variation of viscosity, stenosis height on shearing stress volumetric flow rate are explained, descriptively as well as graphically. The investigation indicates that significant flow changes happen due to the variation of viscosity and stenosis shape.

Keyword: Stenosis height, pressure gradient, wall shear stress, volumetric flow rate, non-Newtonian flow.

### Introduction

The leading cause of deaths in developed and developing countries is a cardiovascular disease. The factors which affect the growth of this type of disease are not yet exactly answered. In circulatory system of the human body highly, oxygenated blood and nutrients deliver from heart to each cell of the body through arteries. Atherosclerosis is a cardiovascular disease which affects arteries by a sub endothelial build-up of fatty or lipid material rich in cholesterol or the accumulation of low density lipoprotein (LDL). Work function is highly affected by the deposition of fatty substances in an artery that's lead to abnormal elongation of arterial thickness which is one of the first smallest step in the formation of atherosclerosis. The accumulation of substances in the artery is known as stenosis and its presence changes various parameters and the flow behavior of blood in the artery. Stenosis may develop at more than one location of arteries . Medical survey shows that stenosis is one the major cause of various cardiovascular diseases Blood flow in the human circulatory system is caused by the pumping action of the heart, which

in turn produces a pressure gradient throughout the system. In the constricted region, formulation of stenosis is normally symmetric about the wall of the artery. As the flow accelerates and consequently the velocity gradient near the wall region is steeper due to the increased core velocity resulting relatively large shear stress on the wall even for a mild stenosis. In recent years, there have been noticed a considerable interest and a good deal of enthusiasm in the viscometric studies and in theoretical modeling of blood flow as well as in the investigation of apparent viscosity, pressure-flow relationship, wall shear stress distribution and resistance to flow, in a vascular channel. Further, proper attention and rapidly growing enthusiasm of researchers, in this particular domain of Biomechanics, have increased quite a lot, with the advent of discovery that many cardiovascular diseases are closely associated with the flow conditions in blood vessels which is clearly associated with CVS diseases. Among the different systems present inside a body, one of the most important systems is obviously cardiovascular system. Human cardiovascular system consists of three primary components viz. (i) Cardio-meaning the heart which is usually a muscular double pump, (ii) Flowing fluid blood which is the most important and a complex physiological fluid and (iii) Blood vessels primarily comprise of different arteries, veins and capillaries, which together, constitute the circulatory network of different pipes. Our heart, ceaselessly pumps blood that flows through blood vessels, supplies nutrition to different organs and tissues, removes metabolic waste products from place to place and regulates physiological conditions in the body. It may be worthwhile in mentioning that an arterial constriction, arising due to an abnormal growth at the lumen of an artery and its gradual advances which is a kind of vascular disease, usually called an atherosclerosis or a stenosis, is closely associated with the changes in blood flow, flow patterns, pressure drop and resistance to flow. The bore of the blood vessels become narrowed due to this unnatural growth in the inner artery wall. This occlusion in a regular flow, arising due to plaque formation at a vessel wall, is reported as the leading cause of mortality in developed countries. Sometimes, for many useful medical and clinical purposes, artificial catheters are inserted in stenosed arteries. Impedance to flow that increases owing to the presence of an unfamiliar growth at the vessel wall, may be further increased due to an insertion of a catheter in an obstructed channel. It may therefore be of interest to investigate the pressure-flow relationship, in a catheterized stenosed artery.

Many researchers have proposed various mathematical models to explain the different features of the blood flow. Mann *et al.* (1938)[4] studied the effects on blood flow of decreasing the lumen of blood vessels, [Young (1968)[12], (1979)[14]; Young and Tsai (1973)[13] analyzed flow characteristics in model of arterial stenosis Caro *et al.* (1978); Shukla *et al.* (1980); Ahmed and

Giddens (1983)[1] studied velocity measurements through axisymmetric stenosis at moderate Reynolds number in steady flow. Sarkar and Jayaraman (1998); Pralhad and Schultz (2004); Jung *et al.* (2004); Liu *et al.* (2004); Srivastava and coworkers (2009)[11] analyzed effects of hematocrit on shear stress and impedance during stenosed artery catheterization. Mishra *et al.* (2006)[5] studied the effects of stenosis and porous parameter on the wall shear stress for the flow of blood in human body. Misra and Verma (2007)[6] studied theoretically characteristics of blood flow through arteries in a pathological state; Ponalagusamy (2007)[9] prepared a two layered model for different types of stenosis and slip velocity at the wall. Layek *et al.* (2009)[3] studied unsteady viscous flow with variable viscosity in a vascular tube with an overlapping stenosis ; Joshi *et al.* (2009)[2] prepared two layered model of blood flow through multi-stenosed artery ; Politis *et al.* (2007)[7], (2008)[8]; prepared numerical model of simulated blood flow in idealized composite arterial coronary grafts in steady state simulations and transient flow Singh *et al.* (2010)[10] studied blood flow characteristics through an artery having radially nonsummetric mild stenosis.

In the present analysis the effect of variation of viscosity and shape of stenosis on the blood flow through stenosis by assuming blood as non-Newtonian (Herschel-Bulkley type) fluid analyzed.

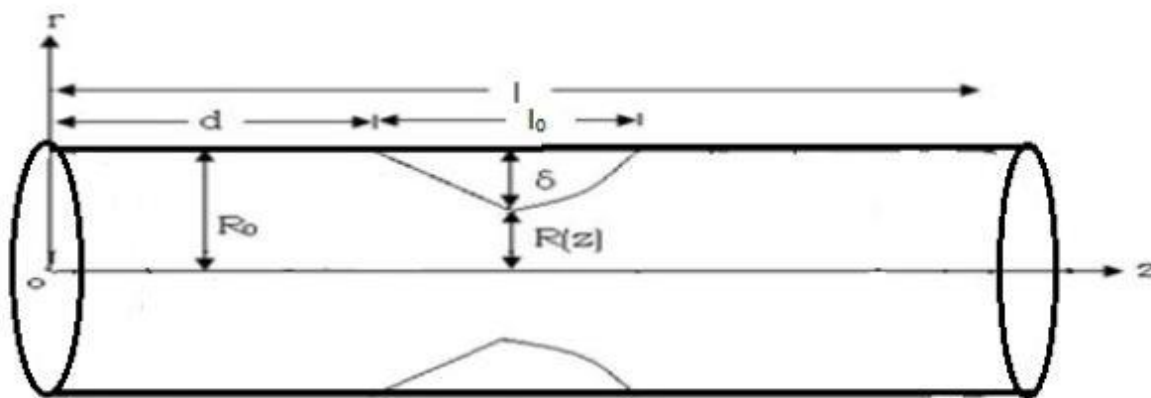


Fig. 1 Geometry of a Stenosed artery

#### Mathematical formulation

Let us consider the steady flow of non-Newtonian blood flow through an mild stenotic artery.

The flow is assumed to be laminar viscous and incompressible.

$$\text{Geometry of the stenosis is } \frac{R(z)}{R} = \begin{cases} 1 - \frac{\delta}{R_0} \left[ 1 + \cos \frac{2\pi}{l_0} \left( z - d - \frac{l_0}{2} \right) \right] & d \leq z \leq d + l_0 \\ 1 & \text{otherwise} \end{cases} \quad (1)$$

The governing equation of motion is

$$-P = \frac{1}{r} \frac{\partial(r\tau)}{\partial r}. \quad (2)$$

Where 
$$-P = \frac{\partial p}{\partial z}$$

r be the radial, z be axial coordinates, p is the pressure and  $\tau$  be the shear stress.

We considered the blood as a non-Newtonian blood, the relation between shear stress and shear rate for this fluid, is given by

$$\tau = \mu \left( -\frac{\partial u}{\partial r} \right)^n + \tau_0 \quad \tau \geq \tau_0. \quad (3)$$

$$\left( -\frac{\partial u}{\partial r} \right) = 0, \quad \tau < \tau_0. \quad (4)$$

Boundary conditions are

$$U=0 \text{ at } r=R \text{ and } R_0 \quad (5)$$

Integrating equation (2) we obtained

$$\frac{-Pr^2}{2} = r\tau$$

$$\tau = -\frac{Pr}{2} \quad (6)$$

From equation (3)

$$\frac{-Pr}{2} = \mu \left( -\frac{\partial u}{\partial r} \right)^{\frac{1}{2}} + \tau_0 \quad \left( \text{For } n = \frac{1}{2} \right)$$

$$-\frac{1}{\mu} \left( \tau_0 + \frac{Pr}{2} \right) = \left( -\frac{\partial u}{\partial r} \right)^{\frac{1}{2}}$$

Squaring and integrating

$$u = -\frac{1}{\mu^2} \left( \tau_0^2 r + \frac{P^2 r^3}{12} + \frac{Pr^2 \tau_0}{4} \right) + A$$

Applying boundary condition

$$u = \frac{1}{\mu^2} \left[ \tau_0(R-r) + \frac{P^2}{12}(R^3 - r^3) + \frac{\tau_0 P}{4}(R^2 - r^2) \right] \quad (7)$$

The volumetric flow rate Q is given by

$$Q = \int_0^{R(z)} u \cdot 2\pi r dr.$$

$$Q = \frac{2\pi}{\mu^2} \int_0^R r \left[ \tau_0(R-r) + \frac{P^2}{12}(R^3-r^3) + \frac{\tau_0 P}{4}(R^2-r^2) \right] dr$$

$$Q = \frac{2\pi}{\mu^2} \left[ \frac{\tau_0 R^3}{6} + \frac{P^2 R^5}{40} + \frac{\tau_0 P R^4}{16} \right]$$

$$Q = \frac{\pi}{\mu^2} \left[ \frac{\tau_0 R^3}{3} + \frac{P^2 R^5}{20} + \frac{\tau_0 P R^4}{8} \right] \tag{8}$$

Wall shear stress is

$$\tau_w = \mu(r) \left( -\frac{\partial u}{\partial r} \right)^{\frac{1}{2}} + \tau_0, \text{ at } r = R(z).$$

Using equation (7)

$$\tau_w = \frac{1}{\mu} \left[ \tau_0 + \frac{P^2 r}{4} + \frac{\tau_0 P r}{2} \right]. \tag{9}$$

## RESULT AND DISCUSSION

The flow characteristics of blood are explained by various analytical expressions and discussed graphically as shown below.

The variation in axial velocity against  $\frac{\delta}{R_0}$  for different values of  $\mu$  is taken into consideration in figure 1. Axial velocity of non-Newtonian blood decrease with increasing of shape parameter and viscosity. It is absorbed that velocity of blood continuously decreases, as stenosis loses its symmetry. Figure 2 Deals with the variation of flow rate (Q) against  $\frac{\delta}{R_0}$  for different values of  $\mu$ . It is observed that the volumetric flow rate remains unchanged in non-stenotic region, as less amount of blood enter into the stenotic region, it reaches its minimum value at the highest value of stenosis height and again starts increasing and reaches to same constant value when blood enters to the non-stenotic region. Figure 3 shows the graphical representations of wall shear stress against radial distance and height of stenosis for different values of  $\mu$ . The wall shear stress(WSS) increases with the increase in radial distance while it decrease with increasing of  $\mu$ .

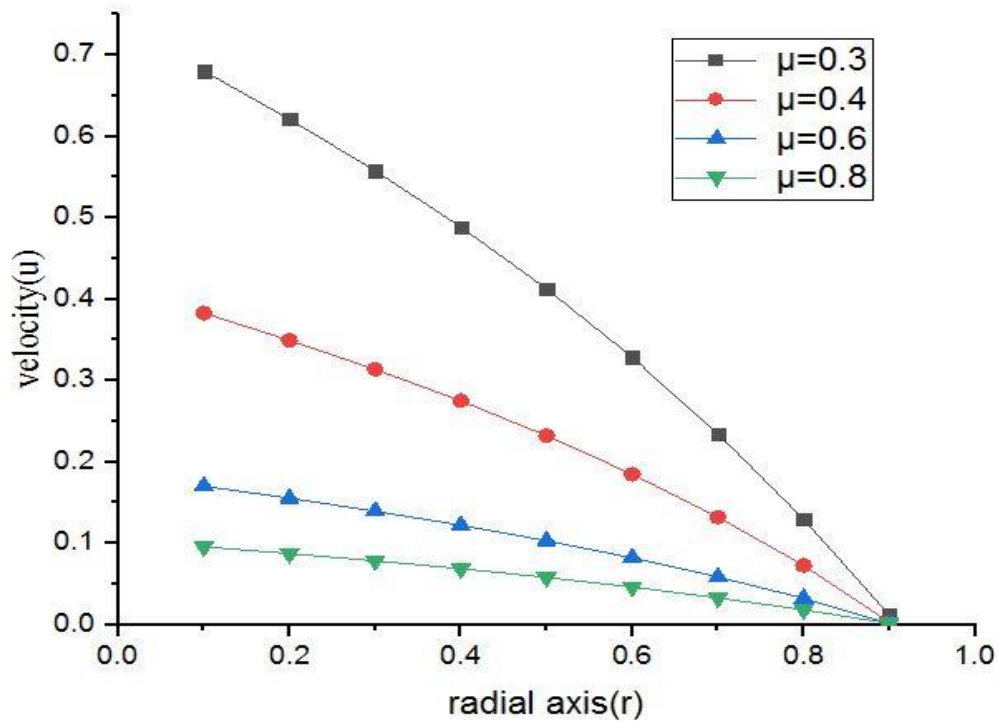


Fig. 1 Variation of velocity (u) against  $\frac{\delta}{R_0}$  for different values of  $\mu$ .

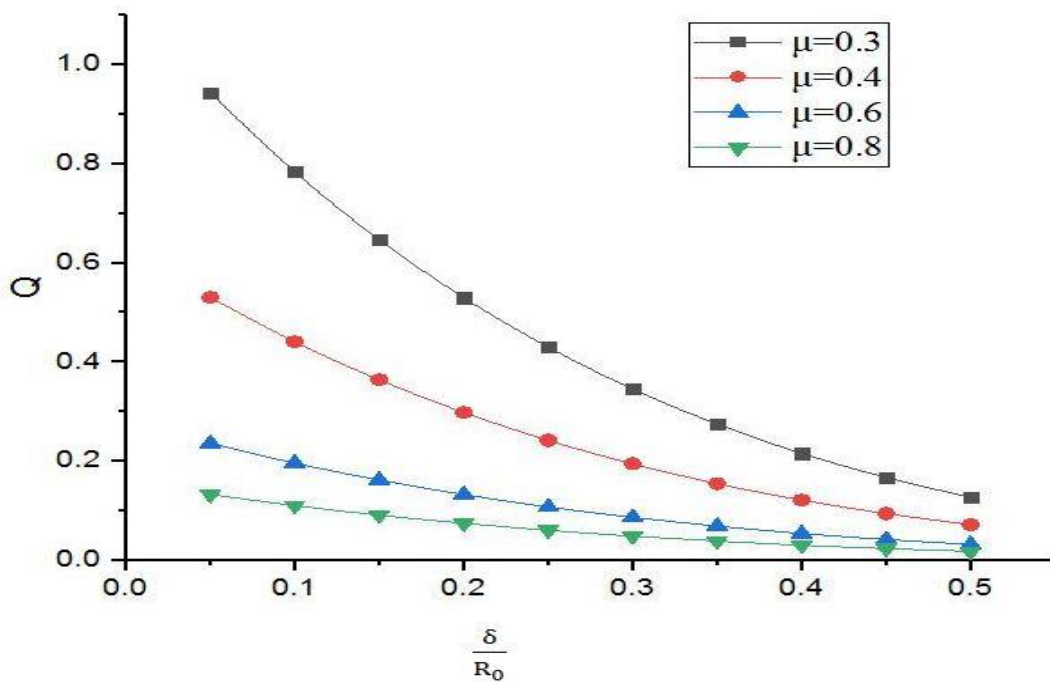


Fig.2 Variation of flow rate (Q) against  $\frac{\delta}{R_0}$  for different values of  $\mu$ .

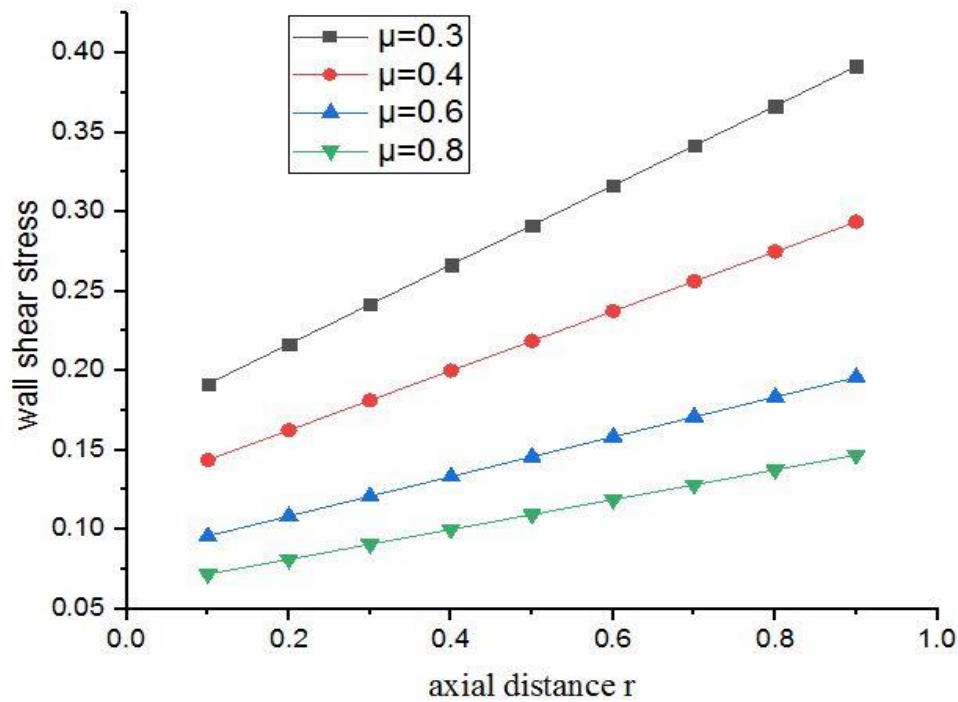


Fig.3 Variation of wall shear stress ( $\tau_w$ ) against  $\frac{\delta}{R_0}$  for different values of  $\mu$ .

### Conclusion

In the present study variation of various parameters like viscosity, stenosis height on shearing stress volumetric flow rate are taken into consideration and discussed thourally. It is found that axial velocity , wall shear stress and volumetric flow rate decreases with increasing of viscosity  $\mu$ . Also the stenosis height affects the shearing stress, wall shear stress and axial velocity. This study can play a important role in the resolution of shear stress, axial velocity and volumetric flow rate in particular situations. These investigation are useful in medical field to treat the cardiovascular disease like hypertension brain stroke patients and to resolve the problems relating the flow of blood in stenotic artery.

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